

Question 1

On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45 \cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \leq t \leq 8$. At the beginning of the workday ($t = 0$), the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \leq t \leq 8$, the plant processes gravel at a constant rate of 100 tons per hour.

- Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.
- Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.
- Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.
- What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

Question 2

A particle moves along a straight line. For $0 \leq t \leq 5$, the velocity of the particle is given by

$$v(t) = -2 + (t^2 + 3t)^{6/5} - t^3, \text{ and the position of the particle is given by } s(t). \text{ It is known that } s(0) = 10.$$

- Find all values of t in the interval $2 \leq t \leq 4$ for which the speed of the particle is 2.
- Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 5$.
- Find all times t in the interval $0 \leq t \leq 5$ at which the particle changes direction. Justify your answer.
- Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

Question 3

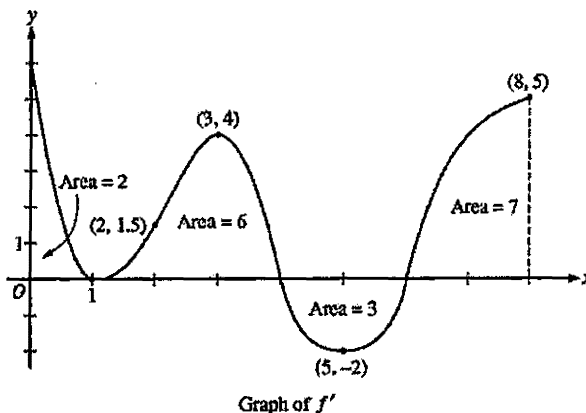
t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t , $0 \leq t \leq 6$, is given by a differentiable function C , where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

- Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.
- Is there a time t , $2 \leq t \leq 4$, at which $C'(t) = 2$? Justify your answer.
- Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6} \int_0^6 C(t) dt$. Using correct units, explain the meaning of $\frac{1}{6} \int_0^6 C(t) dt$ in the context of the problem.
- The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.

Question 4

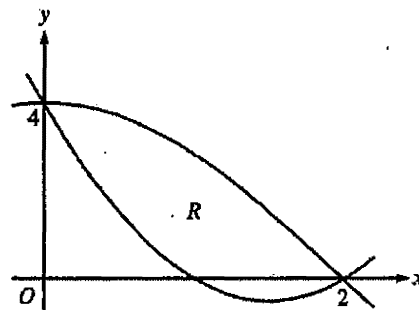
The figure above shows the graph of f' , the derivative of a twice-differentiable function f , on the closed interval $0 \leq x \leq 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x -axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.



- Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.
- Determine the absolute minimum value of f on the closed interval $0 \leq x \leq 8$. Justify your answer.
- On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.
- The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

Question 5

Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region bounded by the graphs of f and g , as shown in the figure above.



- Find the area of R .
- Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.
- The region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

Question 6

Consider the differential equation $\frac{dy}{dx} = e^y(3x^2 - 6x)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through $(1, 0)$.

- Write an equation for the line tangent to the graph of f at the point $(1, 0)$. Use the tangent line to approximate $f(1.2)$.
- Find $y = f(x)$, the particular solution to the differential equation that passes through $(1, 0)$.